Solving Arithmetic Word Problems:

An Analysis of Classification as a Function of Difficulty in Children With and Without Arithmetic LD

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Abstract

This study was designed to determine a word problem difficulty classification in children with arithmetic learning disabilities (ALD; *n* = 104) in comparison with typically achieving students (*n* = 44). We tested variables such as (a) semantic structure (Change, Combine, Compare, and Equalize), (b) operation (subtraction and addition), and (c) position of the unknown quantity in the problem. Facet theory with multidimensional scaling techniques (MINISSA) was used to analyze the underlying dimensions in the responses of each group of participants. Our results indicate that although the word problem difficulty classifications for the 2 groups of children were different, the position of the unknown quantity had a greater influence on the level of difficulty of story problems than other variables. The noncanonical problems—specifically, those with the unknown term in the first place—although difficult for both groups of children, were the most difficult problems for children with ALD.

Since the beginning of the 1980s, the importance of mathematics literacy and problem solving in children's mathematic training has been emphasized (Baroody & Hume, 1991; De Corte, Greer, & Verschaffel, 1996). Until then, math word problems had been used as exercises in the application of the algorithms of addition and subtraction, as it was commonly thought that word problems were difficult for children of all ages and that children should learn to handle the operations of addition and subtraction before attempting to resolve even the simplest of word problems. This was due to the traditional school curriculum, which emphasized only procedural and declarative knowledge, at the expense of conceptual knowledge, and focused on the memorization of facts and computational skills rather than on developing important concepts and applying mathematics to real-world situations (Montague, Warger, & Morgan, 2000).

However, today, instruction that emphasizes reflective thinking and reasoning is considered by many to be critical to mathematics reform (Baroody & Hume, 1991; Jitendra, Di Pipi, & Perron-Jones, 2002; Montague, 1997b). The introduction of word problems in the first years of schooling incorporates mathematics into children's experience and informal education outside school and, thus, facilitates more significant learning of concepts, operations, and arithmetic symbols. Consequently, teachers (National Council of Teachers of Mathematics, 1989) and researchers (e.g., Carpenter & Fennema, 1992; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; De Corte et al., 1996; Patton, Cronin, Bassett, & Koppel, 1997; Montague, 1997a) are insisting on the priority of teaching children how to solve verbal arithmetic word problems, as opposed to algorithms, as a basic didactic principle in the teaching of arithmetic.

A number of studies have found systematic differences in children's word problem–solving performance levels (Judd & Bilsky, 1989; Lewis & Mayer, 1987; Moreno & Mayer, 1999). Judd and Bilsky concluded that comprehension is the most important source of problem difficulty and individual differences in children's mathematical problem performance, because children do not yet have a repertoire of highly automatized schemata for representing the different problem types. Moreno and Mayer (1999) provided evidence that when solving an arithmetic problem, lower achieving students do not benefit from the use of instructional methods that promote the use of multiple representation (i.e., symbolic, visual, and verbal) as opposed to symbolic representation (i.e., only symbolic) in comparison with higher achieving groups of students.

Most models for understanding and assessing children's solution of problems are generally derived from cognitive psychology (Briars & Larkin, 1984; Carpenter & Moser, 1984; Kintsch & Greeno, 1985; Riley, Greeno, & Heller, 1983). Based on these models, a number of studies have demonstrated that semantic structure is much more relevant than syntax in studying children's solutions of addition and subtraction problems (Carpenter & Moser, 1982; Carpenter, Hiebert, & Moser, 1981). Carpenter and Moser (1983) proposed the following classification of word problems as a function of semantic structure: Change, Combine, Compare, and Equalize. In the *Change* problems, there is an initial quantity and a direct or implied action that causes an increase or decrease in that quantity. For example, "Pablo had 18 stickers. His friend Juan gave him 6 more stickers. How many stickers does Pablo have altogether?" *Combine* problems involve the static relationship existing between a particular set and its two disjoint subsets: "There are 12 sheep in a van; 4 are black, and the rest are white. How many white sheep are there?" *Compare* problems also involve a static relationship in which there is a comparison of two distinct, disjoint sets: "Olivia's bicycle has 14 gears, and Alba's bicycle has 9 gears. How many less gears does Alba's bicycle have than Olivia's?" Finally, in *Equalize* problems, there is the same sort of action as found in Change problems but based on the comparison of two disjoint sets: "My dress has 12 buttons. If my sister's dress has 5 buttons more, it will have the same number of buttons as my dress. How many buttons does my sister's dress have?" Moreover, each of these word problem types, though having the same semantic structure, varies in difficulty depending on which value in the problem is the unknown quantity.

In addition to the various semantic relations, there are other ways in which the problems differ. Taking into account the work of Riley et al. (1983), there are three items of information in each kind of problem. In Change problems, these are the start, change, and result sets. Any of these can be found if the other two are given (i.e., the unknown quantity may be the start, the change, or the result), so there are a total of six kinds of Change problems. In Compare problems, where the direction of the difference may be more or less, and the unknown quantity may be the amount of difference between the referent set and the compared set or either of the two sets themselves, a total of six variations are also possible. There are fewer possible variations in Combine problems; the unknown quantity is either the combined set or one of the subsets. In Equalize problems, the unknown quantity may be only in the difference between the given quantity and the desired quantity, although a total of six variations are possible. Therefore, following Riley et al. (1983), "the idea that quantitative change, equalization, combination, and comparison emerge at different times in cognitive development is too simplistic. To understand children's problem-solving skills, the identity of the unknown quantity must also be taken into account" (p. 161).

Different classifications of the difficulty of story problems have been found across diverse grades: Tamburino (1980) in preschool; Carpenter, Hiebert, and Moser (1981) in preschool, first, second, and third primary grades; or Riley (1981) in first grade. Maza (1990) synthesized the main results of these studies, organizing the problems in ascending order of difficulty: (a) Combine 1, Change 1 and 2; (b) Compare 1, Change 3 and 4; (c) Compare 2, 3, and 4, Combine 2, and Change 5 and 6; and finally (d) Compare 5 and 6 —. If we examine these schemata more closely, the problems vary at different levels, depending on which value in the problem is unknown. In a first analysis level of this variable, it seems that problems are easier when the unknown is situated in the result, independently of the kind of presented problem.

In another study, Riley et al. (1983) found that the difficulty of Change problems increases when the unknown quantity is the starting set instead of the change or the result set. Moreover, children's performance clearly decreases in additive Combine and Compare problems when the unknown is one of the addends. This difficulty further increases when the value of the first addend is unknown. In subtraction situations, Change problems are easier than Combine problems, and these in turn are easier than Compare problems. Furthermore, when the minuend is unknown, the level of difficulty increases in each of the word problem categories.

Carpenter and Moser (1982, 1983, 1984) and Carpenter (1985) confirmed that the most difficult word problems are those that cannot be represented directly by models (e.g., changes where the initial quantity is the unknown). Lack of knowledge of the initial quantity causes children to attempt erroneous processes to solve the problem. Story problems that cannot be easily modeled are significantly more difficult than those that can be. Similar results were found by Gibb (1956) and Lindvall and Ibarra (1980).

Solving arithmetic word problems (SAWP) has been considered an important area of deficit in children with arithmetic learning disabilities (ALD; Mercer & Miller, 1992; Miller & Mercer, 1997; Russell & Ginsburg, 1984). Students with ALD experience considerable difficulty either with problem representation or in identifying relevant information, along with difficulties in reading, computation, and identifying operations (Parmar, Cawley, & Frazita, 1996). Furthermore, Jordan and Montani (1997) studied calculation and problem solving in two subgroups of children with ALD: one group with specific difficulties in mathematics, and another with deficits in reading as well as mathematics.

Their results confirmed that the difficulties of children with specific math deficits are related to the retrieval of number facts from memory, whereas the group with more general difficulties had deficits that were more associated with the conceptualization and resolving of problems. Fuchs and Fuchs (2002) described the mathematical problem–solving profiles of students with mathematics disabilities (MD) with and without comorbid reading disabilities (RD); the differences between students with MD only and those with MD + RD were mediated by the level of problem solving (arithmetic story problems vs. complex story problems vs. real-world problem solving) and by performance dimension (operations vs. problem solving). On arithmetic story problems, the differences between the disability subtypes were similar for operations and problem solving. In contrast, on complex story problems and real-world problem solving, the differences between the subtypes were larger for problem solving than for operations.

The SAWP ability of children with low mathematics performance is particularly affected by the semantic structure of the problem and the identity of the unknown quantity. Jiménez and García (1999) found that ALD students and students with garden-variety (GV) math difficulties are equally affected by these variables. So children with ALD and those with GV poor mathematics performance show similar patterns of performance in solving arithmetic word problems. These results suggest that both variables have more influence on individuals with poor mathematics performance than on typically achieving children. In another study, García and Jiménez (2000) showed that the identity of the unknown quantity plays an important role in explaining the relative difficulty of word problems for children with ALD and those with GV poor mathematics performance. Both groups had more difficulties with solving noncanonical than canonical problems (see Note 1) in spite of their semantic structure. Therefore, the aim of the present study is twofold:

1. to analyze how the performance of children with ALD compares to that of typically achieving (TA) students, investigating the effects on children's ability to SAWP of variables such as (a) semantic structure (Change, Combine, Compare, or Equalize), (b) the operation (subtraction or addition), and (c) the unknown value in the problem (canonical or noncanonical); and
2. to determine in a detailed manner the classification of word problem difficulty for both groups in order to facilitate more appropriate sequencing of word problem instruction in the school and in intervention programs.

Method

Participants

A sample of 148 Spanish children was studied. These children came from urban areas and from average socioeconomic backgrounds, were attending several state schools, and ranged in age from 7 years 1 month to 9 years 4 months (M = 7.81; SD = 0.67). Children with mathematics difficulties were defined as those who had scores below the 25th percentile on the *Batería de Aptitudes Diferenciales y Generales* (BADYG; Yusté, 1985; n = 104; age M = 7.8, SD = 0.6). In this group were included children with arithmetic learning disabilities (ALD; 24 boys, 36 girls) as well as children with GV poor mathematics performance (22 boys, 22 girls). We must clarify that although sample differentiation based on the IQ–achievement discrepancy criterion (see Note 2) was carried out for a previous study, in this study we did not consider it useful, as we had shown that there were no significant differences in achievement level between discrepant and nondiscrepant children when they had to solve arithmetic word problems (see García & Jiménez, 2000; Jiménez & García, 1999, 2002). Therefore, in this study, the participants will be treated as a single group of students with arithmetic learning disabilities. The remainder of the children were defined as typical achievers (TA; 15 boys, 29 girls) because they scored above the 30th percentile on the BADYG Arithmetic subtest (n = 44; age M = 7.9, SD = 0.7) and their score on an IQ test was above 80. There were significant statistical differences in IQ between both groups, $t(146) = 8.418$, p = .000. There were no significant statistical differences in the distribution of the participants as a function of gender, $\chi^2(2, 148) = 2.36$, p = .30. Moreover, the results showed that there were no differences between the groups in age, $t(146) = -1.65$, p = .065. Neither was there a significant effect of age in solving arithmetic word problems, $t(146) = 1.13$, p = .26. Children with arithmetic difficulties were receiving some educational support in the resource classroom for a few hours a week.

Procedure

Teachers nominated children who had specific difficulties with mathematics but did not have a history of reading problems, children who had difficulties in all subject areas, and children who showed average performance in all subjects. We selected the children who were typically achieving (TA group) and the children with poor mathematics performance based on the results of the BADYG Arithmetic subtest (ALD group).

The children completed the problems in three sessions of 20 min each. They were tested individually in a quiet room, and the order in which the word problems were presented was counterbalanced. The children had the written problems in front of them the whole time. Also, each participant was read each word problem, and his or her task was to carry out any actual arithmetic operations and then tell the examiner how he or she had solved the problem. Participants were instructed to listen to the single auditory presentation of each problem, and they could make notes while the examiner read each word problem. No time limitations were imposed. The participants were asked to solve 40 word problems, preceded by four practice problems to familiarize them with the task. Counters, paper, and pencils were offered to the children by the examiner.

Instruments and Materials

Math Measure. The Arithmetic subtest of the *Batería de Aptitudes Diferenciales y Generales* (BADYG; Yusté, 1985) consists of a written section with increasingly difficult computations. Level 2 (ages 6 to 7) consists of 35 items, each of which has four alternative responses, to assess basic addition and subtraction operations. Level 3 (ages 8 to 9) consists of 32 items, each of which has five alternative responses, to assess calculation speed, numerical judgment, and operation in logical–numerical problems. A reliability analysis using the split-half procedure gave a coefficient of .86 at Level 2 and .91 at Level 3, and the correlation between BADYG Arithmetic subtest score and class grade was .53, $p < .01$, for Level 2, and .44, $p < .01$, for Level 3.

Test of Intelligence. The *Wechsler Intelligence Scale for Children–Revised* (WISC-R; Wechsler, 1989) was administered. In a reliability analysis using the split-half procedure for the Spanish revision of the WISC-R, the correlation coefficient (Spearman-Brown) was .93.

Arithmetic Word Problems. Overall, two items were designed for each of 20 arithmetic word problem categories, yielding a total of 40 problems. The categories of Change, Combine, Equalize, and Compare are representative of categorical schemes that have been used by several investigators (e.g., Carpenter & Moser, 1982) in analyses of simple addition and subtraction problems. As mentioned previously, in addition to the various semantic relations, word problems differ depending on which quantity is unknown; thus, the number of possible problems could be increased. (The Appendix shows examples of the several kinds of word problems used in this study.) Sentence length, syntactic complexity, and vocabulary difficulty were controlled while the arithmetic word problems were designed. The quantity magnitude was also controlled, as all the word problems always included combinations of units and tens. Solutions to the word problems were considered to be correct when the child carried out counting procedures correctly and there was no operation error (for details of psychometric properties, see Jiménez & García, 1999).

Design

To test the hypothesis that distinguishes three facets (i.e., semantic structure, operation involved, and position of the unknown quantity) in the structure of arithmetic word problems for each group of participants (TA group and ALD group), we proposed a *facet theory* design and analysis. This research strategy is based on a structural framework that includes design and analysis of data, beginning with a generic approach to the object of study. Evidence of the adjustment of empirical data to a determined theoretical structure is searched for, repeating the process with different samples of participants and variables to consolidate a model (Borg & Shye, 1995; Canter, 1983; Donald, 1995). This process indicates that an optimum strategy in constructing a theory is to hypothesize a correspondence between the definition of a certain behavioral domain and the structure of the empirical observations of the variables chosen to represent the aforementioned domain. These internal structures are found using various techniques that analyze the proximity of stimuli, fixing the stimuli structure in a dimensional space known as the spatial solution. This spatial solution is determined by the similarity of variables, in that the more alike two variables are, the nearer they will be in space. In brief, the aim is to empirically test a given structure on a semantic level through a pattern of similarities between variables.

To apply facet theory techniques, a model of the content universe must be devised using a formal and detailed definition. This definition includes the facets (in our research, semantic structure, operation involved, and position of the unknown quantity) that, according to its elements, should classify the variables that represent the domain of interest. A mapping sentence comprising three facets with four, two, and three elements (see Figure 1) was then set up.

Due to the four elements of Facet A (semantic structure; i.e., Change, Combine, Compare, and Equalize), we expect a quantitative distinction (axial role) between the items as a function of the semantic structure of the word problem. The elements of Facet B (operation involved) establish a quanti-

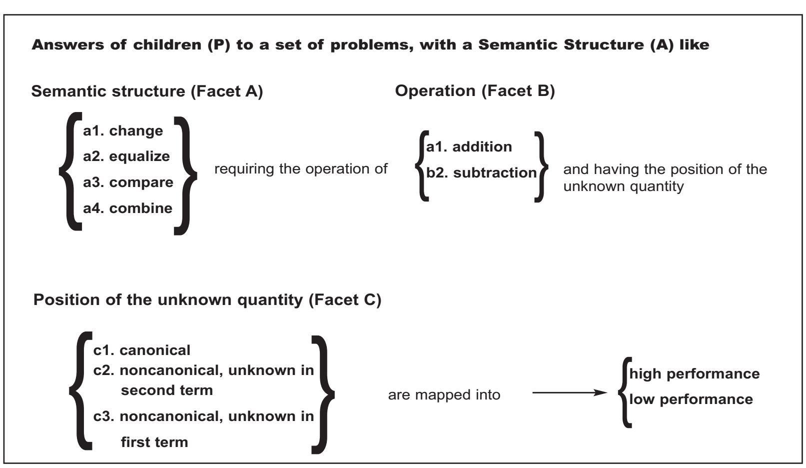

FIGURE 1. Mapping sentence.

tative distinction (axial role) between items whose solution requires addition and those items requiring subtraction. The elements of Facet C (position of the unknown quantity) establish a third quantitative difference (axial role) between the items in terms of the semantic structure being canonical (the unknown quantity is the operation's result); noncanonical with the unknown quantity in the second term (c2); or noncanonical with the unknown in the first term (c3). The assignment of facet elements to the items was not exhaustive; for instance, there were no word problems that at the same time required addition and were canonical (see structuples of each item in Table 3).

Results

We began by calculating a sum score for each of the 20 arithmetic word problem categories. Tables 1 and 2 show the means of the sum scores in word problems for both groups (TA and ALD). To exclude the possibility that intelligence was modulating achievement, we carried out a MANCOVA, obtaining a nonsignificant effect of IQ as covariate, $F(20, 125) = 1{,}128$, $p = .331$.

To facilitate the identification of the underlying dimensions in the responses, the sum scores of the correct solutions were analyzed using the multidimensional scaling program ALSCAL (SPSS, v, 11-) for each group of participants. The multidimensional scaling was carried out on the matrix of indices of item proximity. Three-dimensional solutions were retained for both groups, due to the increment in the stress index for a two-dimensional solution and due to the three proposed facets. The solutions produced a Kruskall stress index of 0.213 ($R \leq = 0.9256$) for the TA group and 0.086 ($R \leq = 0.953$) for the ALD group.

Figures 2, 3, and 4 present the obtained spatial solution for the TA group, showing Dimensions 1 and 2, Dimensions 1 and 3, and Dimensions 2 and 3, respectively, and Figures 5, 6, and 7 present the spatial solution for the ALD group. In both solutions (TA and ALD), Dimension 1 corresponded to the degree of difficulty of word problems—that is, items that are located in higher spatial regions have higher means of sum scores of correct solutions (see Tables 1 and 2) than those located in lower spatial regions.

Each facet was tested on each possible bidimensional space. As we will see, all facets that allow separations fulfill the hypothesized axial roles of separation—that is, reveal quantitative rather than qualitative classifications.

Figure 2 is a combined codification of Facets B and C. Items are marked with a double code: on the one hand in terms of the required operation (Facet B), with a square for addition and a circle for subtraction, and on the other hand in terms of the position of the unknown quantity (Facet C); the canonical problems (c1) are marked in black, the noncanonical problems with the unknown quantity in first place (c3) are marked in gray, and the noncanonical problems with the unknown quantity in second place (c2) are marked in white. The classification capacity of these two facets is clear. The degree of difficulty (Dimension 1) is closely related to the position of the unknown quantity; the canonical items with the unknown quantity in the op-

TABLE 1
Means and Standard Deviations of Sum Scores for Typically Achieving Children

Item type[a]	*M*	*SD*
Equalize 4	0.4091	0.7569
Compare 6	0.7955	0.9042
Change 6	1.2045	0.8782
Equalize 5	1.2045	0.8782
Compare 5	1.3636	0.8096
Change 3	1.5227	0.7921
Compare 1	1.5227	0.7921
Equalize 3	1.5227	0.7622
Compare 2	1.5682	0.6954
Equalize 1	1.6591	0.7134
Change 5	1.7273	0.6943
Equalize 2	1.7727	0.6048
Change 1	1.7955	0.4615
Combine 2	1.7955	0.5938
Equalize 6	1.7955	0.4615
Compare 3	1.8182	0.4952
Compare 4	1.8182	0.4952
Change 4	1.8864	0.3868
Combine 1	1.9091	0.3621
Change 2	1.9318	0.2550

Note. Listwise valid $N = 44$; range = 0.00–2.00 for all items except Change 2 (1.00–2.00).
[a] See Appendix for definition and examples.

TABLE 2
Means and Standard Deviations of Sum Scores for Children with Arithmetic Learning Disabilities

Item type[a]	*M*	*SD*
Compare 5	0.3654	0.6394
Equalize 5	0.3750	0.6710
Equalize 4	0.4135	0.6626
Compare 1	0.4231	0.6638
Change 5	0.4904	0.7632
Compare 6	0.5192	0.7880
Change 3	0.5769	0.8325
Equalize 1	0.5865	0.7583
Change 6	0.6058	0.7685
Combine 2	0.6827	0.8275
Compare 4	0.7596	0.8184
Equalize 2	0.8269	0.8640
Equalize 6	0.9038	0.8305
Compare 2	0.9135	0.8138
Change 4	0.9423	0.8570
Change 2	1.0962	0.8760
Compare 3	1.1346	0.8709
Equalize 3	1.3269	0.8296
Combine 1	1.3462	0.7343
Change 1	1.4904	0.7240

Note. Listwise valid $N = 104$; range = 0.00–2.00 for all items.
[a] See Appendix for definition and examples.

eration's result (c1; black) have the highest sum scores and are located in upper spatial regions, and the non-canonical items with the unknown term in first place (c3; white), except Change 5, are those with the lowest sum scores and are located in lower spatial regions, c3 subtraction items being easier than c3 addition items.

On Dimension 2, there is a clear separation in terms of the required operation. In lower coordinates (at left) are located items that involve subtraction (circles), and in upper coordinates (at right) are the items that involve addition (squares). Only two subtraction items—Combine 2 and Comparison 4—are slightly displaced toward the addition area. The operation involved does not seem to be related with Dimension 1; we find subtraction items in upper as well as in lower regions.

In Figure 3 (Dimensions 2 and 3), the classification of Facet B (operation) on Dimension 2 can be seen again. The classification proposed by semantic structure (Facet A) is not reflected either here or in Figure 4.

However, it would seem (see Figure 4) that Change and Combine task types are mostly located in the spatial regions of easy problems—the upper region of Dimension 1. The difficulty cannot be related to operation—the two most difficult items (Compare 6 and Equal 4) require addition—but on high sum scores there is no separation due to operation.

The figures representing the spatial solution obtained for the ALD group follow patterns similar to those described for the TA group. In Figure 5 (Dimensions 1 and 2), Facet C (position of the unknown quantity) can be seen on Dimension 1, associating again the difficulty level with the unknown quantity position. However, in the ALD group, the distribution of the problems in terms of difficulty (see Tables 1 and 2) differs from the distribution of the TA group, as they are more scattered toward the difficult levels and there are far fewer items with high scores of correct solutions. Again, the c3 problems (unknown term in first

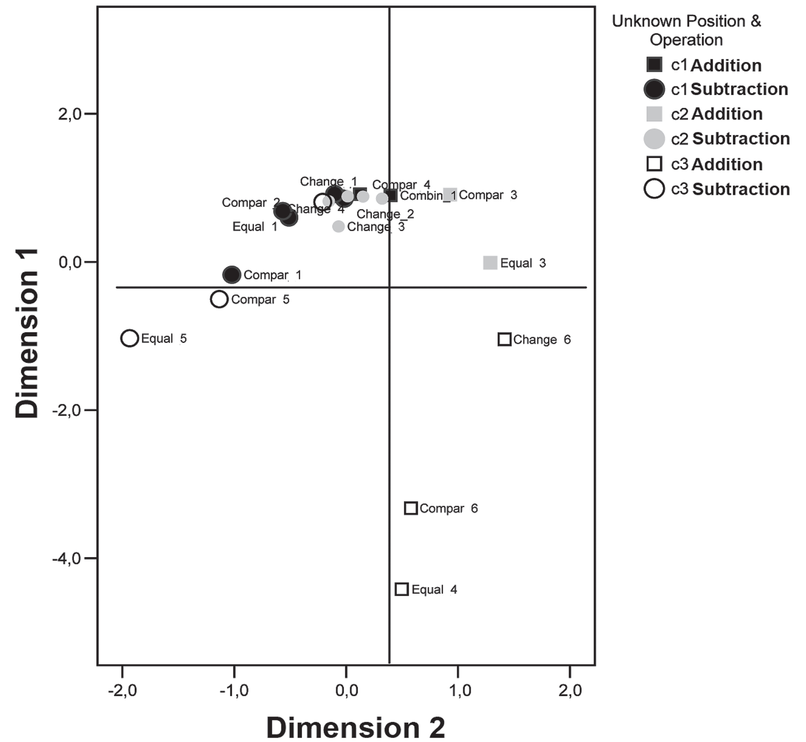

FIGURE 2. Facets B and C on the spatial solution for Dimensions 1 and 2 for the group of typically achieving children.

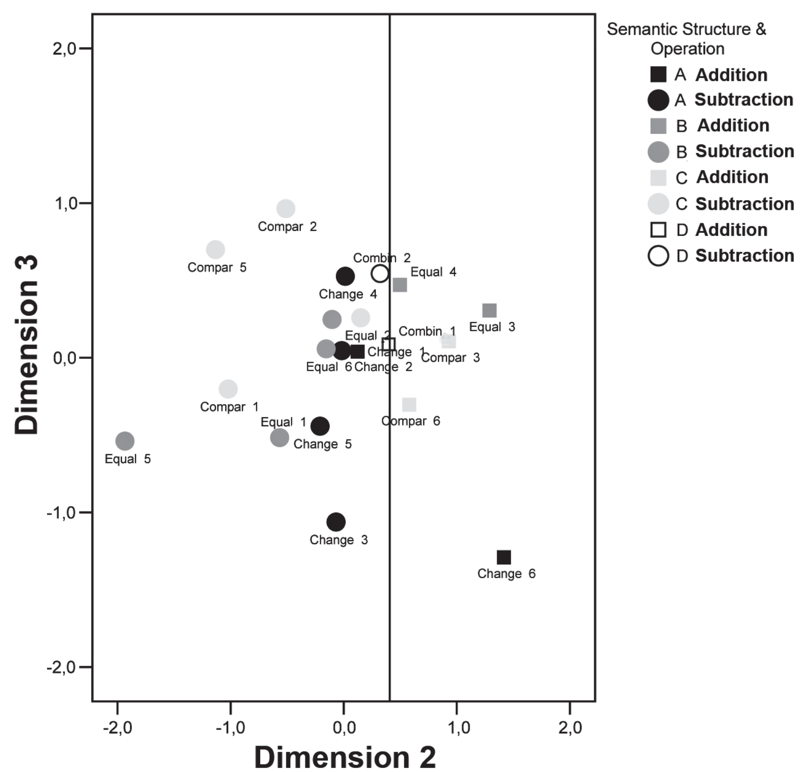

FIGURE 3. Facet B on the spatial solution for Dimensions 2 and 3 for the group of typically achieving children.

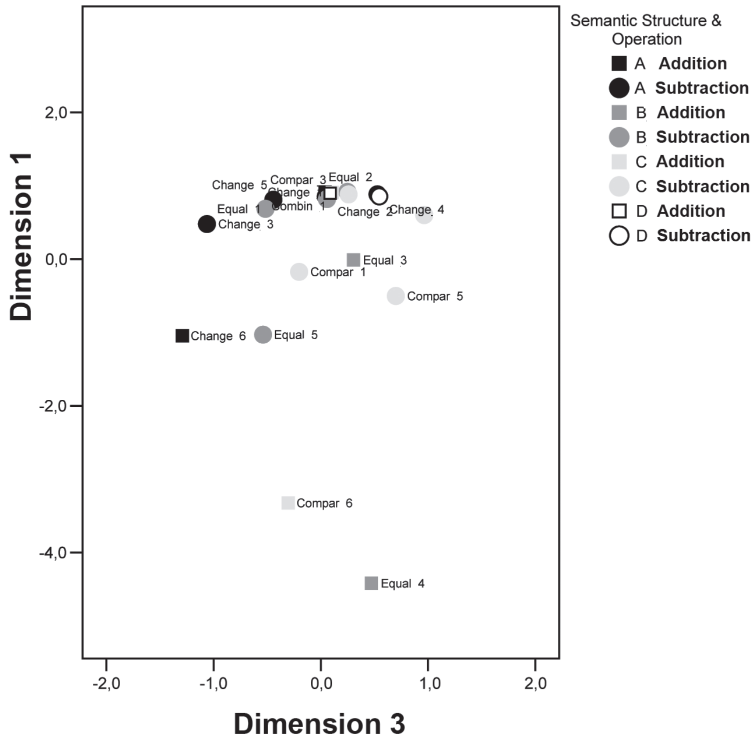

FIGURE 4. Spatial solution for Dimensions 1 and 3 for the group of typically achieving children.

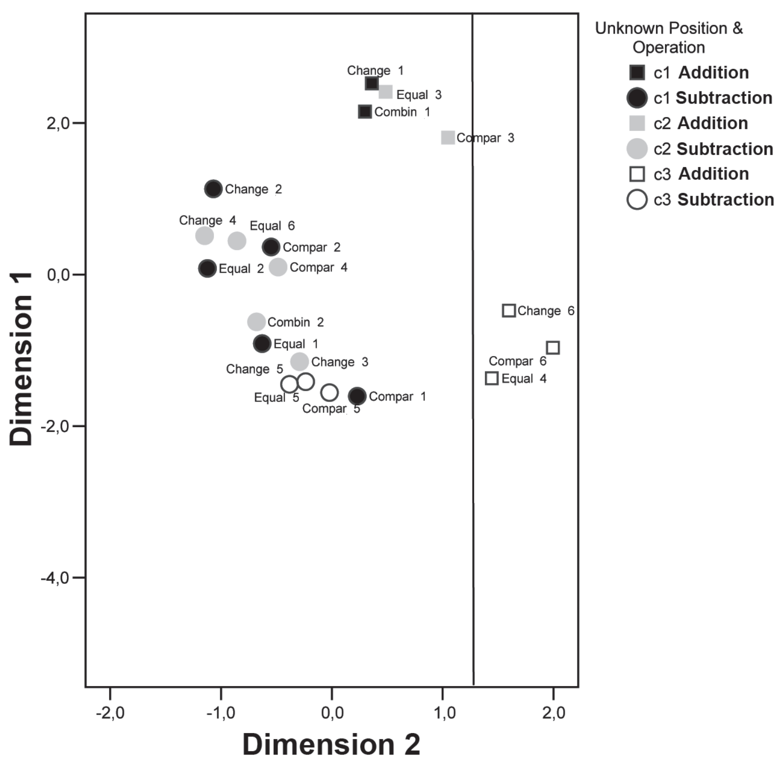

FIGURE 5. Facet B on the spatial solution for Dimensions 1 and 2 for the group of children with arithmetic learning disabilities..

place; white) are the most difficult, but for this group, c3 addition is not easier than c3 subtraction. Also, we find in the lower region the item Comparison 1, which in this sample had much lower scores of correct solutions. On Dimension 2, we find the same clear division due to operation (addition or subtraction), but in this group we can see that the c1 and c2 addition problems (gray and black squares) are the easiest. Subtraction problems are distributed throughout the medium- and high-difficulty regions.

In Figure 6 (Dimensions 2 and 3), the same classification of Facet B (operation) on Dimension 2 can be seen. On Dimension 3, due to Facet A (semantic structure), the hypothesized classification separates the items at least in terms of the first element (i.e., the Change tasks), in contrast with the TA sample, where no separation could be found. All Change problems have low coordinates, in contrast to the other problems, which are distributed throughout this dimension. In Figure 7 (Dimensions 1 and 3), there does not seem to be a definite pattern combining the difficulty of the problem with the semantic structure.

Discussion

The aim of this study was to determine word problem difficulty classifications in students with arithmetic learning disabilities in comparison with typically achieving students using facet theory analysis. Our results showed that semantic structure by itself was not enough to determine the degree of difficulty of the problems, although Change problems were appreciably different from the other categories. This probably occurs because, according to Carpenter (1985), Change is the only type of problem that occurs over time, and this can have an influence on children's performance.

Our findings are supported by previous studies (Bermejo, Lago, & Rodríguez, 1998; Carpenter, Hiebert, & Moser, 1981; Riley et al., 1983; Riley &

Greeno, 1988) indicating that the *position of the unknown quantity* has a greater influence on the level of difficulty of word problems than other variables. The results show that non-canonical problems—specifically, those with the unknown term in the first place (i.e., start in Change, referent in Compare, making equal the known set in Equalize)—are those where children show a lower number of correct responses. This variable affects children with arithmetic learning disabilities more than typically achieving students. Thus, the items that are easier for typically achieving children are moderately difficult for students with arithmetic learning disabilities, and those that are moderately difficulty for the more skilled group mostly become high-difficulty items for students with low achievement.

Although, in general, addition problems are easier, the difficulty of word problems is caused not so much by the operation (addition or subtraction) demanded by the problem as by the position of the unknown term. So, as several studies have shown (i.e., Carpenter & Moser, 1983; Grouws, 1972; Lindvall & Ibarra, 1980), problems with the unknown term in first place that require subtraction are more difficult for typically achieving children than those requiring addition. There are no differences between these two types of problems for students with arithmetic learning disabilities; in this case, subtraction and addition present the same difficulty level.

With regard to the facet theory approach, two of the three hypothesized facets became visible in both samples. Facet B (operation involved) classified items clearly. Facet C (position of the unknown quantity) caused separation according to the degree of difficulty. Facet A (semantic structure) only appeared partially in the group of children with ALD, but not in TA children. Thus, it would be useful to replicate this study with other samples of children and other samples of items with the purpose of consolidating the graduated classification obtained.

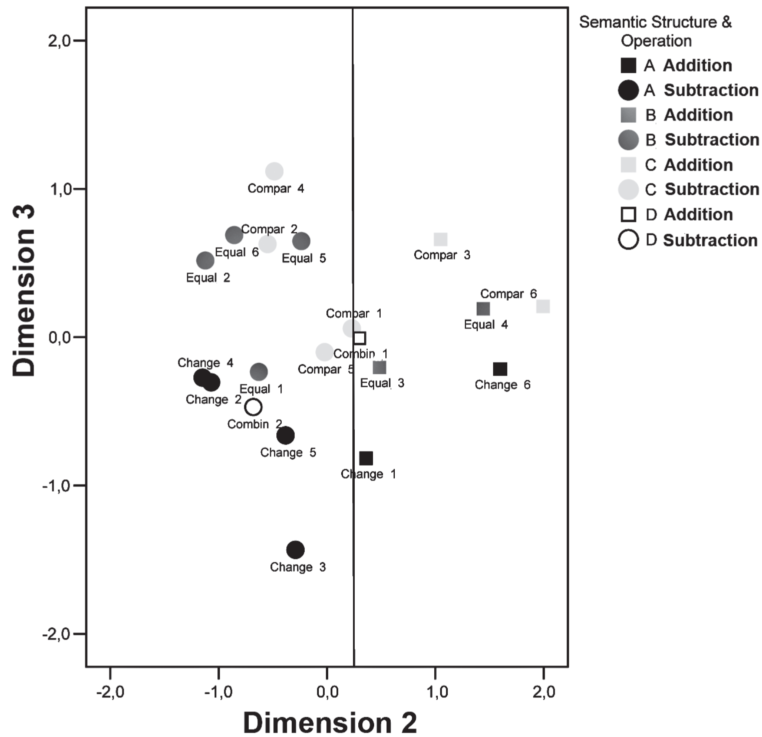

FIGURE 6. Facet B on the spatial for Dimensions 2 and 3 for the group of children with arithmetic learning disabilities.

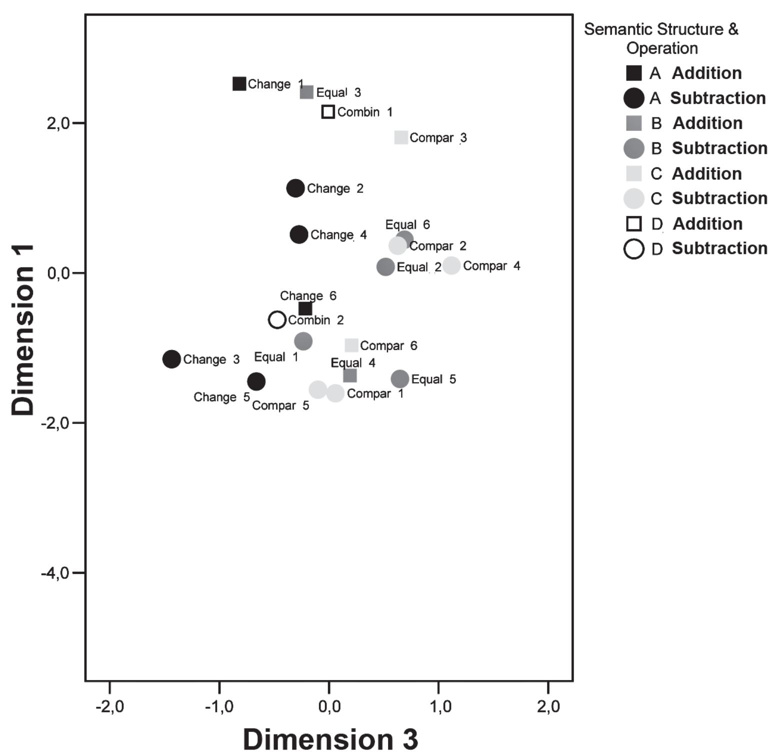

FIGURE 7. Spatial solution for Dimensions 1 and 3 for the group of children with arithmetic learning disabilities.

In summary, the most significant findings of this study suggest that problems with unknown terms in the first place are more difficult for children with arithmetic learning disabilities than for typically achieving children. The relative difficulty of word problems is clearly different for these two groups of students. The performance of children with arithmetic learning disabilities is one or even two levels below that of their classmates. The order of difficulty can be established in three levels, from less difficult to more difficult (see Table 3).

Taking into account the suggestions of Riley and Greeno (1988) and Riley et al. (1983), we suggest that children with poor mathematics performance do not possess schemata that represent these semantic relations and connect them to solution sequences. According to Riley et al. (1983), the acquisition of problem-solving skills is primarily an improvement in children's ability to represent the relationships among quantities described in the problem situation. According to Jitendra, Di Pipi, and Perron-Jones (2002), knowledge of the mathematical structure of problems can facilitate the activation of the relevant schemata or patterns that would guide problem representation, which is necessary for solving problems. Schemata are the basis for understanding the appropriate mechanism for the problem solver to capture both the patterns of relations and their linkages to operations (Mar-

TABLE 3
Order of Word Problem Difficulty in Three Levels by Math Ability Group

Low difficulty			Moderate difficulty			High difficulty		
Problem[a]	**P*x***	**Op**	**Problem[a]**	**P*x***	**Op**	**Problem[a]**	**P*x***	**Op**
			TA Group					
Change 2	c1	–	Compare 1	c1	–	Compare 6	c3	+
Combine 1	c2	+	Compare 5	c3	–	Equalize 4	c3	+
Change 4	c2	–	Equalize 5	c3	–			
Compare 3	c2	+	Change 6	c3	+			
Compare 4	c2	–						
Change 1	c1	+						
Combine 2	c2	–						
Equalize 6	c2	–						
Equalize 2	c1	–						
Change 5	c3	–						
Equalize 1	c1	–						
Compare 2	c1	–						
Equalize 3	c2	+						
Change 3	c2	–						
			ALD Group					
Change 1	c1	+	Change 2	c1	–	Change 3	c2	–
Combine 1	c2	+	Change 4	c2	–	Compare 6	c3	+
Equalize 3	c2	+	Compare 2	c1	–	Change 5	c3	–
Compare 3	c2	+	Equalize 6	c2	–	Compare 1	c1	–
			Equalize 2	c1	–	Equalize 4	c3	+
			Compare 4	c2	–	Equalize 5	c3	–
			Combine 2	c2	–	Compare 5	c3	–
			Change 6	c3	+			
			Equalize 1	c1	–			

Note. P*x* = position of unknown quantity (Facet C); Op = operation (Facet B); TA = typically achieving children; ALD = children with arithmetic learning disabilities.
[a]See Appendix for definition and examples.

shall, 1995). In fact, "the schema-based strategy would allow students with poor memory abilities to organize information using semantic relations and thus to acquire adequate word problem solving skills" (Jitendra & Hoff, 1996, p. 430). These same authors demonstrated that students with ALD could improve their performance in mathematical word problem solving when instruction emphasized activities that teach both conceptual understanding and efficient execution of processes and strategies.

Taking into consideration our results and the suggestions offered by Ginsburg (1997), we consider that the content of curricula should be sequenced according to the developmental and cognitive characteristics of the students—and not in terms of the logical structure of mathematics. Knowledge of the variables involved in the difficulty of the problems is extremely important and has significant educational implications for early math assessment and instruction, particularly for children with difficulties in mathematics. Further research is necessary to assess the positive effects of this word problem instructional sequence in intervention programs for children with ALD. Our findings cannot be generalized to other word problems, such as those with several steps or multiplication and division problems. Future research should extend this study to new mathematical domains.

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NOTES

1. *Math word problems can be defined as canonical when the unknown quantity is the target operation's result. They are non-canonical when the unknown quantity is the first or second term of the operation.*
2. *Children were classified as having ALD if their arithmetic standard score was more than 15 points lower than their IQ score (N = 60) and their score on an IQ test was > 80. Children were considered to have GV poor mathematics performance if their arithmetic score was less than 15 points lower than their IQ score (N = 44) and their score on an IQ test was > 80. Children who had sensory deficits, acquired neurological deficits, or other problems traditionally used as exclusionary criteria for ALD were excluded.*

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APPENDIX
Examples of Problems Used

Change

Change 1. Join. Result unknown:
Antonio had 18 stickers. His friend Bentenuya gave him 6 more stickers. How many stickers does Antonio have altogether?

Change 2. Separate. Result unknown:
Zebenzuí had 14 coins. He gave 3 coins to Juan. How many coins has he got left?

Change 3. Join. Change unknown:
Sara had 5 diamonds in her bracelet. Then Alba gave her some more diamonds. Now there are 12 diamonds in Sara's bracelet. How many diamonds did Alba give her?

Change 4. Separate. Change unknown:
Pablo's mother made 10 muffins. Then she gave some of the muffins to her neighbor. Now Pablo has 7 muffins. How many muffins did Pablo's mother give her neighbor?

Change 5. Join. Start unknown:
My fishbowl had some fishes. Then I put in 4 fishes more. Now I have 12 fishes. How many fishes did I have at the beginning?

Change 6. Separate. Start unknown:
There were some frogs in a small lake. After 16 frogs leapt away, there were 7 left. How many frogs were there in the beginning?

Compare

Compare 1. Direction of difference *more than.* Difference unknown:
Gara has 12 balls. Irene has 5 balls. How many more balls does Gara have than Irene?

Compare 2. Direction of difference *less than.* Difference unknown:
Olivia's bicycle has 14 gears, and Anita's bicycle has 9 gears. How many gears does Anita's bicycle have less than Olivia's?

Compare 3. Direction of difference *more than.* Compared quantity unknown:
Berto bought a pencil that cost 12 pennies and a notebook that cost 9 pennies more than the pencil. How much did the notebook cost?

Compare 4. Direction of difference *less than.* Compared quantity unknown:
My uncle has 17 pairs of shoes, and my grandmother has 8 pairs less than my uncle. How many pairs of shoes does my grandmother have?

Compare 5. Direction of difference *more than.* Referent unknown:
Sandra has 11 cousins. She has 3 more cousins than Yaiza. How many cousins does Yaiza have?

Compare 6. Direction of difference *less than.* Referent unknown:
Emilio's sweater has 19 buttons. His sweater has 5 buttons less than Berto's sweater. How many buttons does Berto's sweater have?

Equalize

Equalize 1. Join. Equalizing value unknown:
Juanito has been in the school football team for 13 years, and his brother Francisco for 8 years in the same team. How many years does Francisco have to stay in the team to have been there as many years as Juanito?

Equalize 2. Separate. Equalizing value unknown:
Hector has 14 toys, and Romen has 5 toys. How many toys does Hector have to take away to have as many toys as Romen?

Equalize 3. Join. To make the known set equal:
Marcelo has 15 pennies. If his mother gives him 9 more, he will have the same number as David. How many pennies does David have?

Equalize 4. Separate. To make the unknown set equal:
On the bus that goes to La Laguna, there are 17 people; if 6 people get off the bus to Santa Cruz, there will be the same number of people on it as on the bus that goes to La Laguna -. How many people are there on the bus to Santa Cruz?

Equalize 5. Join. To make the unknown set equal:
My dress has 12 buttons. If my sister's dress has 5 buttons more, it will have the same number of buttons as my dress. How many buttons does my sister's dress have?

Equalize 6. Separate. To make the known set equal:
Nico has 13 pyjama jackets. If he gives 9 away, he will have the same number of pyjama jackets as Izem. How many pyjama jackets does Izem have?

Combine

Combine 1. Combined value unknown:
Your father has 14 uncles and your mother has 5. How many uncles do they have altogether?

Combine 2. Subset unknown:
There are 12 sheep in a van; 4 are black, and the rest are white. How many white sheep are there?